

1. The vertices of an equilateral triangle with side lengths 6 lie on the sides (one per side) of an equilateral triangle with side lengths 10. A point is chosen at random inside the larger triangle. Find the exact probability that the chosen point also lies inside the smaller triangle.

2. A jar contains 15 quarters, 20 dimes, 10 nickels and 5 pennies. A monkey picks four coins at random without replacement. What is the probability that the total value of the drawn coins is at least 30 cents? Round your answer to the nearest thousandth.

3. Box I contains 3 Red and 5 Blue balls. Box II contains 1 Red and 7 Blue balls. Box III contains 6 Red and 2 Blue balls. An integer n is chosen at random from 1 through 10. If n is a square, a ball is removed from Box I. If n is prime, a ball is removed from Box II. Otherwise, a ball is removed from Box III. If the chosen ball is Red, what is the exact probability that it came from Box III?

4. In a standard deck of 52 cards, twos through tens are worth 2 points; Jacks, Queens and Kings are worth 5 points; Aces are worth 10 points. Two cards are drawn without replacement. What is the expected total point value of the drawn cards? Round your answer to the nearest thousandth.

5. A student is to draw 8 cards, with replacement, from a standard deck of 52. The student wins a prize if he/she draws at least one Ace. If five students perform this experiment, what is the probability that at least two will win a prize? Round your answer to the nearest thousandth.

1. The probability is the ratio of the areas, or $\left(\frac{6}{10}\right)^2 = \frac{9}{25}$. Report $\frac{9}{25}$ or .36.

2. The probability of at least 30 is $1 - P(\text{less than } 30)$.

The following (Q, D, N, P) ordered quadruples give a total less than 30:

$(1, 0, 0, 3), (0, 2, 1, 1), (0, 2, 0, 2), (0, 1, 3, 0), (0, 1, 2, 1), (0, 1, 1, 2), (0, 1, 0, 3),$
 $(0, 0, 4, 0), (0, 0, 3, 1), (0, 0, 2, 2), (0, 0, 1, 3), (0, 0, 0, 4)$.

The first of these can happen in $C(15,1) \cdot C(5,3) = 150$ ways. The second can happen in $C(20,2) \cdot C(10,1) \cdot C(5,1) = 9500$ ways. The remaining outcomes happen in, respectively, 1900, 2400, 4500, 2000, 200, 210, 600, 450, 100, 5 ways. Add these and divide by $C(50,4)$ to get .09559 and subtract from 1 to get .904.

3. $P(R) = \frac{3}{10} \cdot \frac{3}{8} + \frac{4}{10} \cdot \frac{1}{8} + \frac{3}{10} \cdot \frac{6}{8} = \frac{31}{80}$. Bayes' Theorem gives the following:

$$P(\text{III} | R) = \frac{P(R | \text{III}) \cdot P(\text{III})}{P(R)} = \frac{(6/8)(3/10)}{(31/80)} = \frac{18}{31}$$

4. It is hardly obvious that the expected point values on the two draws are the same, namely, $2 \cdot \frac{36}{52} + 5 \cdot \frac{12}{52} + 10 \cdot \frac{4}{52} = \frac{43}{13}$, giving the answer $(2)(3.3077) = 6.615$, so refer to the tree diagram on the next page.

The possible point totals and their probabilities are given below:

| | | | | | |
|-------------------------------------|---|-------------------------------------|--|--|-----------------------------------|
| 4 | 7 | 10 | 12 | 15 | 20 |
| $\frac{36}{52} \cdot \frac{35}{51}$ | $2 \cdot \frac{36}{52} \cdot \frac{12}{51}$ | $\frac{12}{52} \cdot \frac{11}{51}$ | $2 \cdot \frac{36}{52} \cdot \frac{4}{51}$ | $2 \cdot \frac{12}{52} \cdot \frac{4}{51}$ | $\frac{4}{52} \cdot \frac{3}{51}$ |

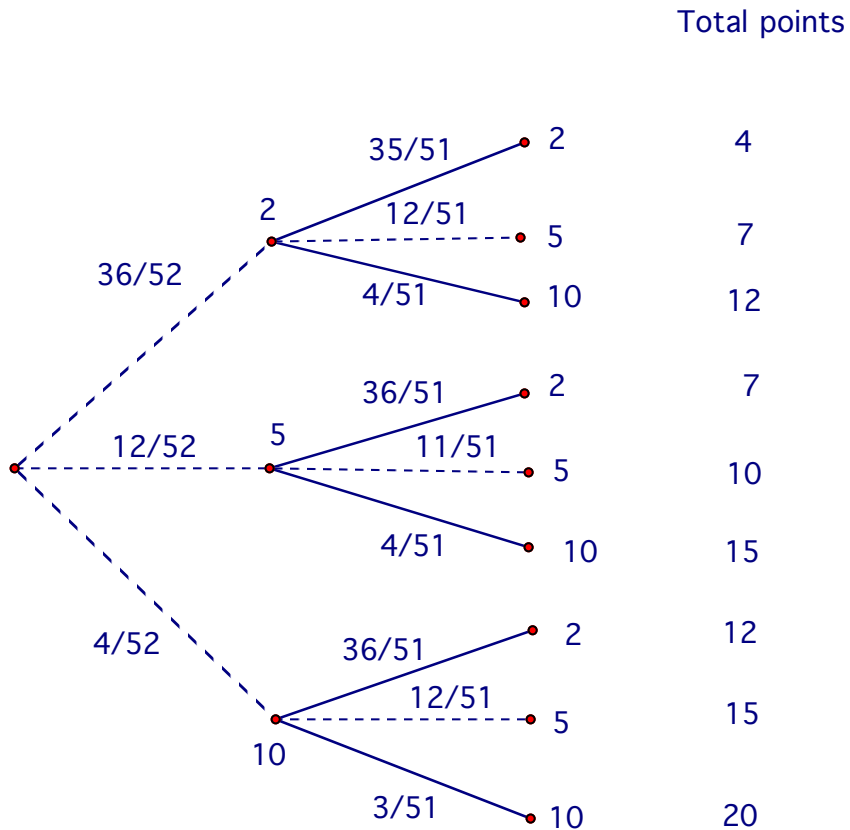
To find the expected value, multiply each of the six values by its probability and add to get 6.615.

5. The probability that a student wins a prize is

$1 - P(\text{no Aces}) = 1 - (48/52)^8 = .4728877$. The probability of failure is .5271123.

The probability of at least two winning students is

$$1 - P(0 \text{ or } 1 \text{ winner}) = 1 - (.5271123)^5 - C(5,1)(.5271123)^4(.4728877) = .777$$



Examples:

$$P(7 \text{ points}) = (36/52)(12/51) + (12/52)(36/51)$$

$$\begin{aligned}
 P(5 \text{ on second draw}) &= P(2, 5) + P(5, 5) + P(10, 5) \quad [\text{dashed paths}] \\
 &= (36/52)(12/51) + (12/52)(11/51) + (4/52)(12/51) \\
 &= (12/52)(36/51 + 11/51 + 4/51) = (12/52)(51/51) = 12/52 = P(5 \text{ on first}), \\
 &\text{leading to Expected value of second draw} = \text{Expected value of first draw.}
 \end{aligned}$$

North Suburban Mathematics League
Meet 2 – Seniors - Probability
October 28, 2010

ANSWERS

1. $\frac{9}{25}$ or .36
2. .904
3. $\frac{18}{31}$
4. 6.615
5. .777